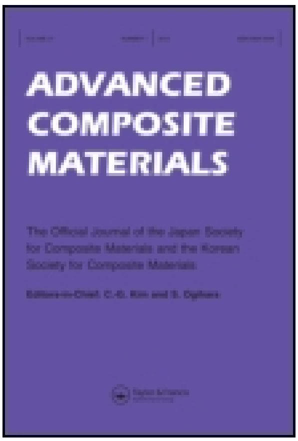

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Experimental evaluation of effective tensile properties of laminated composites

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Abstract—Laminated structures are normally designed using a set of strength values that are determined experimentally. Such values are determined by carrying out experiments on unidirectional test coupons. In this paper it is argued that the predicting the failure of multidirectional laminates based on the lamina to laminate approach cannot be accurately done using lamina properties obtained from unidirectional coupon test. Instead, one needs to obtain the effective properties of laminae as deduced from multidirectional laminate test. A method is therefore proposed to determine the effective tensile properties of a composite laminate. In particular, two types of composite materials were examined, namely carbon fiber-reinforced epoxy and glass fiber-reinforced epoxy. The failure behavior of laminates made from these materials is explained and the effect of multiple cracking in strength reduction is outlined. The experimental results confirm that the failure of multidirectional laminates cannot be accurately predicted using unidirectional laminae properties. Instead, the paper describes how to obtain the effective properties and shows that these are quite different from those obtained using unidirectional configurations.

Keywords: Effective properties; composite; laminates; testing; failure.

1. INTRODUCTION

Classical lamina-to-laminate procedures for predicting the strength of laminated composites have been well documented [1, 2]. While minor differences exist between the various proposed theories, they all have one feature in common: that is, laminate failure is dictated by the failure of its individual laminae as they experience certain limit stresses. Actual lamina stresses used for this comparison are computed using the classical lamination theory (CLT), which also calculates the laminate stiffness properties, based on the lamina stiffness properties. Both the lamina elastic

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properties and experimental strength values are obtained from a set of standard tests that are mostly performed on unidirectional, uniaxial specimens [3].

While yielding accurate estimates of the laminate stiffness properties, the analysis procedure described above has quite often resulted in erroneous strength prediction. Different reasons have been given to explain these discrepancies. Some of these reasons relate to the inability of the CLT to account for the out-of-plane stresses that may develop between differently oriented laminae within multidirectional laminates. These stresses can become very significant around discontinuities and traction free edges such as those encountered in straight edge test coupons. This phenomenon is often referred to as the edge effect [4, 5].

The incapability of the CLT to account for out-of-plane stresses cannot be considered as the only reason for its lack of accuracy in predicting laminates strength. Another equally important reason has to do with the difference between the elastic and failure properties of laminae when present in a multidirectional laminate and when they are considered as unidirectional entities. The properties of individual laminae bonded together to form a laminate, sometimes called *in-situ* properties, can be quite different from the unidirectional lamina properties. Reasons for such differences have been adequately discussed in the literature [6]. Here, it suffices to mention that one major reason has to do with the change in crack growth patterns in a given lamina when angle-plied or cross-plied with other laminae. This change is likely to have a direct influence on matrix-dominated properties (transverse and shear properties).

Since the accuracy of lamina-to-laminate strength analysis depends greatly on its input parameters, namely the lamina properties, it becomes very important to properly determine representative lamina properties. To offset the disadvantages of unidirectional test coupons some authors [6, 7] have recommended the use of a number of multidirectional specimen configurations to obtain the various 'effective' properties required to fully characterize fiber-reinforced lamina. For example, the standard 0° unidirectional specimen used for obtaining tensile properties of fibre-reinforced laminae according to ASTM Test Method D3039-76 is deemed inappropriate to obtain any property but, possibly, the longitudinal Young's modulus E_1 and the associated Poisson's ratio ν_{12}. A number of alternative specimens and rules of thumb are thus proposed to obtain the 'effective' properties of the lamina. The common feature among the proposed test specimens is that the target lamina (lamina for which the properties are sought) be constrained in one way or the other with adjacent lamina, thus ensuring a more accurate prediction of the properties intended for use with the CLT within the context of a lamina to laminate failure analysis.

These new testing philosophies and 'rules of thumb' for determining lamina properties discussed in Refs [6, 7] were formulated with advanced aerospace composites in mind, the kind composed of very strong and stiff fibers in a very brittle matrix. As such, new general guidelines were proposed that only apply to this class of materials. For example, it was proposed that the transverse stiffness of a unidirectional lamina be reduced by a factor of three, while the transverse strength be increased by a factor of about two. Experimental results reported in this paper will be compared with such rules and with published standard (ASTM) lamina properties. The principal subject of this paper is therefore, to propose specific procedures for determining those lamina properties that cannot be accurately determined from standard unidirectional tests and also to limit the use of rules of thumb. Ultimately, the aim of this work is to show how these new 'effective' properties can be used to accurately predict the strength of any multidirectional composite laminate under static and possibly fatigue loads. First, the new tests are described and the procedures for deducing the effective lamina properties from these tests are outlined. An experimental investigation is then carried out to compare the properties obtained using the new procedures with those obtained using standard test methods and to quantify the improvement in laminate strength predictions when these effective properties are used. Finally, advantages associated with the new test methods are delineated and some recommendations are formulated.

2. NEW TEST AND DATA REDUCTION PROCEDURES

2.1. *Transverse Young's modulus* E_2

It has been argued effectively in Ref. [6] that there is a need to replace the existing procedure to obtain the transverse Young's modulus E_2. Here, it is proposed that cross-plied average laminate's longitudinal modulus (E_x) be experimentally measured. E_x can also be evaluated from a simple rule of mixture that incorporates the volume ratios of the various laminae that make up the laminate. In a cross-plied laminate with a volume ratio r_1 of 0° laminae and a volume ratio r_2 of 90° laminae E_x is approximated by

$$E_x = r_1 E_1 + r_2 E_2, \tag{1}$$

where E_x is the longitudinal modulus of the cross-plied specimen, E_1 is the longitudinal modulus of the 0° lamina, r_1 is the ratio of the volume of all the 0° laminae to the total volume of the specimen, r_2 is the ratio of the volume of all the 90° laminae to the total volume of the specimen.

The standard specimen proposed here is an eight layer $(0°, 90°)_{2s}$ 25.4 mm wide coupon. Using cross-plied laminates and knowing that the laminate is made up of equal number of 90° and 0° layers, equation (1) yields the following simple expression for evaluating E_2:

$$E_2 = 2E_x - E_1. \tag{2}$$

This is purely based on the assumption that the volume fractions of plies are equal. To validate this assumption, the classical lamination theory was utilized to calculate the transverse modulus E_2 from average laminate modulus E_x and 0° longitudinal E_1. Using the following relationships:

$$E_x = \frac{A_{11}A_{22} - A_{12}^2}{tA_{22}}, \tag{3}$$

$$\nu_{21} = \frac{\nu_{12}E_2}{E_1}, \tag{4}$$

$$A_{ij} = \sum_{k=1}^{n} (Q_{ij})_k (h_k - h_{k-1}). \tag{5}$$

These equations are well documented [1–4].

2.2. Transverse tensile strength Y

There is a considerable difference between the transverse tensile strength of an unconstrained unidirectional 90° lamina and that of a constrained one. This has been explained in many studies; small initial microcracks in a 90° lamina will propagate and cause rapid failure unless arrested by fibers in adjacent layers, which is what happens in a multidirectional laminate. Obviously, the strength of the 90° lamina alone becomes of little usefulness and should be replaced by the effective strength of the constrained lamina. As mentioned in Ref. [6], such a practice has already been adopted by at least two different organizations. These organizations opted to constrain the 90° laminae with 0° laminae from a more ductile material or with ±45° laminae from the same material. In this paper we propose that the same specimen, cross-plied, be used for evaluating the transverse tensile strength Y. A simple procedure to obtain this property would be as follows:

(a) Subject the specimen to a monotonic tensile load.

(b) Monitor the longitudinal modulus E_x given by σ_x/ε_x.

(c) Record the membrane load (N/mm or lb/in.) at the point where the laminate longitudinal modulus σ_x/ε_x drops to the modulus of an equivalent laminate with negligible modulus for the 90° laminae, i.e.: $E_x = r_1 E_1$.

The recorded load would then be used in conjunction with the classical lamination theory to determine the stress in the 90° lamina:

$$N = \int_{-t}^{t} \sigma \,\mathrm{d}z \qquad \text{Membrane loading,} \tag{6}$$

$$\varepsilon_x = \frac{\partial u^o}{\partial x} - z\frac{\partial^2 w}{\partial x^2} = \varepsilon_x^o + z\kappa_x \qquad \text{Strain-displacement equation,} \tag{7}$$

where t in equation (6) is the thickness of the laminae. Combining equations (6), (7) and the transformed stiffness matrix gives:

$$\{N\} = \sum_{k=1}^{m} \left(\int_{z_{k-1}}^{z_k} [Q]^k \,\mathrm{d}z \right) \{\varepsilon^o\} + \sum_{k=1}^{m} \left(\int_{z_{k-1}}^{z_k} [Q]^k z \,\mathrm{d}z \right) \{\kappa\}. \tag{8}$$

In a balanced symmetric laminate and for the in-plane membrane forces, the equation (8) is reduced to:

$$\{N\} = [A]\{\varepsilon^o\} + [B]\{\kappa\}. \quad (9)$$

The strain obtained from equation (9) is then used to determine the stress in the target lamina.

$$\{\varepsilon\}_x = \{\varepsilon^o\}_x + z\{\kappa\}_x, \quad (10)$$

$$\{\sigma\}_x = \{Q\}_x\{\varepsilon\}_x. \quad (11)$$

This stress is considered as the effective transverse failure stress Y of a unidirectional lamina. Stress determined in the 90° lamina based on the above described procedure is not purely uniaxial. In fact, in a cross-plied laminate made from typical advanced composite laminae, a compressive stress component will develop in the 90° lamina parallel to the fibers. The ongoing analysis is based on the assumption that the effect of such a longitudinal compressive stress on the effective transverse strength of a lamina can be neglected.

2.3. Longitudinal tensile strength X

The $(0°, 90°)_{2s}$ specimen will again be proposed to obtain the longitudinal tensile strength of fiber-reinforced laminae. This value is in fact obtained in a straightforward manner by loading the specimen to failure and simply dividing the failure load by the total cross-section of the 0° laminae. This derives from the assumption that the 90° laminae would have completely failed prior to the ultimate failure of the specimen. Hence, they do not contribute to the structural strength of the laminate. In fact, it is argued here that the contribution of 90° layers is a functional one in that they act as arresters to any initial cracks that exist in the 0° lamina. The assumption that the 90° layers would have failed prior to ultimate failure also signifies that the stress state in the target 0° layer is purely uniaxial. This further justifies the use of a simple load over area stress calculation.

3. EXPERIMENTAL CORRELATION

3.1. Sample preparation

To determine the various effective properties described in the previous section and to correlate the findings with common types of multidirectional laminates close to 50 specimens were manufactured from two different fiber-reinforced materials, namely, carbon/epoxy type AS4-3501 and glass/epoxy type 1003. These materials will be referred to as AS4 and 1003 throughout this work.

Two plates of 254 mm by 114 mm were laid up according to the $(0°, 90°)_{2s}$ configuration and cured in an autoclave under controlled temperature and pressure according to the pre-preg manufacturer's guidelines. Each plate was then cut to obtain specimens that are 25.4 mm wide by 254 mm long. To reduce the possibility of failure at the gripping section each test specimen was tabbed using aluminum 25.4 mm by 63.5 mm by 6 mm. The samples were monotonically loaded to failure using an electro-mechanical universal testing machine. The load was measured by a 267 kN load cell and the strain was measured by a 50.8 mm gauge length extensometer made by MTS. The accuracy of both load cell and extensometer were in conformity with ASTM regulation, which specifies 1% accuracy. The applied strain rate was also with regard to ASTM testing standards of 0.2% strain per minute.

3.2. Experimental data for AS4

The AS4 cross-plied $(0^\circ, 90^\circ)_{2s}$ laminate was loaded to first determine the longitudinal modulus E_x. Figure 1 shows a plot of σ_x *vs.* ε_x. The longitudinal Young's modulus was determined using the linear elastic stress–strain relationship at the first ply failure stress. The method of least square was used to determine E_x. In addition to cross-plied, eight samples of 0° unidirectional laminates were tested to failure and E_1 was determined in similar manner. Figure 2 depicts the σ_x *vs.* ε_x of 0° unidirectional laminate.

Edge replicas were used to replicate the pattern on the edge of the sample on a film where the formation of any crack can later be examined. The result of edge-replica procedure showed that the first crack in the 90° layer occurred at the laminate stresses of 152 MPa. However, no significant change in modulus was detected up to 276 MPa. This was therefore, selected as the failure stress limit of the laminate when complete failure of 90° layer occurs. Then 24 specimens were tested to 276 MPa stress to determine laminate E_x. Using equation (2), and knowing the

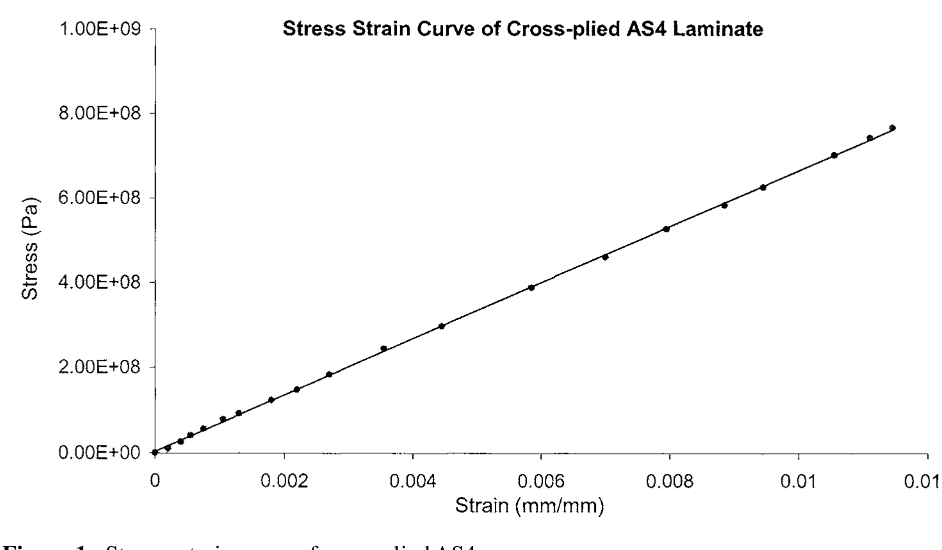

Figure 1. Stress–strain curve of cross-plied AS4.

volume fraction of 0° and 90° is 50% each, $V_{90}^{o} = V_{0}^{o} = 50\%$, the transverse modulus was determined. At the same time, using CLT equations, the average longitudinal modulus of laminate and equations (3)–(5), transverse modulus was also determined. The results are summarized below.

The experimental data are as follows:

$E_x = 70.8$ GPa,

$E_1 = 125.4$ GPa,

$E_2 = 16.2$ GPa Effective using the rule of mixture,

$E_2 = 15.5$ GPa Effective using CLT and utilizing equations (3)–(5),

$E_2 = 9.5$ GPa Unidirectional (by testing unidirectional coupons).

Clearly, the result of the rule of mixture is in good agreement with the result of classical lamination theory, with approximately 4.5% difference. To further test these results, both transverse moduli obtained from the rule of mixture (R of M) and CLT were used to predict the failure strength of cross-plied, angle-plied and quasi-isotropic laminates. The predicted values for these laminates are listed in Table 1.

The ultimate strength of 0° unidirectional was experimentally determined to be 1950 MPa. Table 1 summarizes the ultimate longitudinal strength of the cross-plied, angle-plied and quasi-isotropic laminates. These values were found using the guidelines outlined in the test method section. These tests were carried out to check the validity of the assumption made earlier. It was assumed that once the off-axis plies completely fail, the only layer contributing to the strength of the laminate is the 0° laminate. Edge replica process was also performed on all

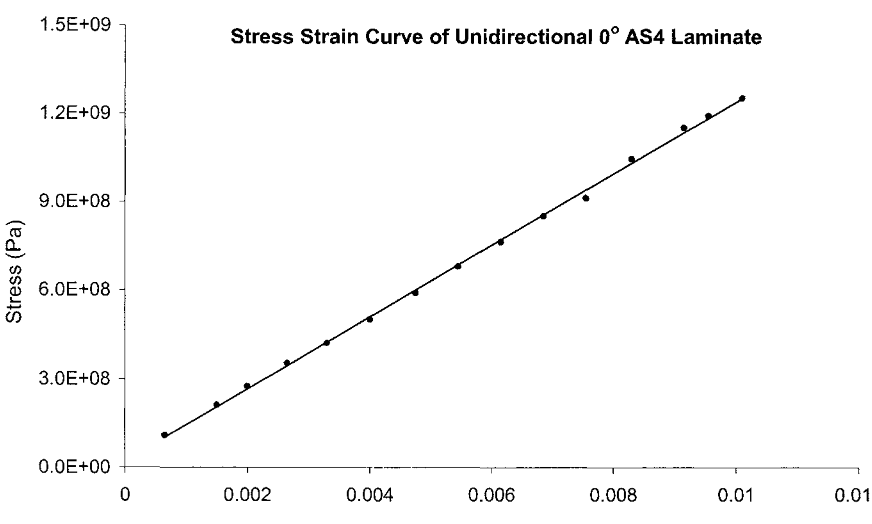

Figure 2. Stress–strain curve of 0° undirectional AS4.

Table 1.
Effective strengths X and Y of AS4

	X (Cross-plied) $(0^\circ, 90^\circ)_{2s}$	X (Angle-plied) $(0, \pm 45, 0)_s$	X (Quasi-isotropic) $(0, \pm 45, 90)_s$	Effective strength Y
Experimental value	1510 MPa	1550 MPa	1830 MPa	50.0 MPa unidirectional (90)
R of M prediction	1485 MPa	1493 MPa	1547 MPa	62.7 MPa multi-directional
CLT prediction	1482 MPa	1486 MPa	1539 MPa	60.0 MPa multi-directional

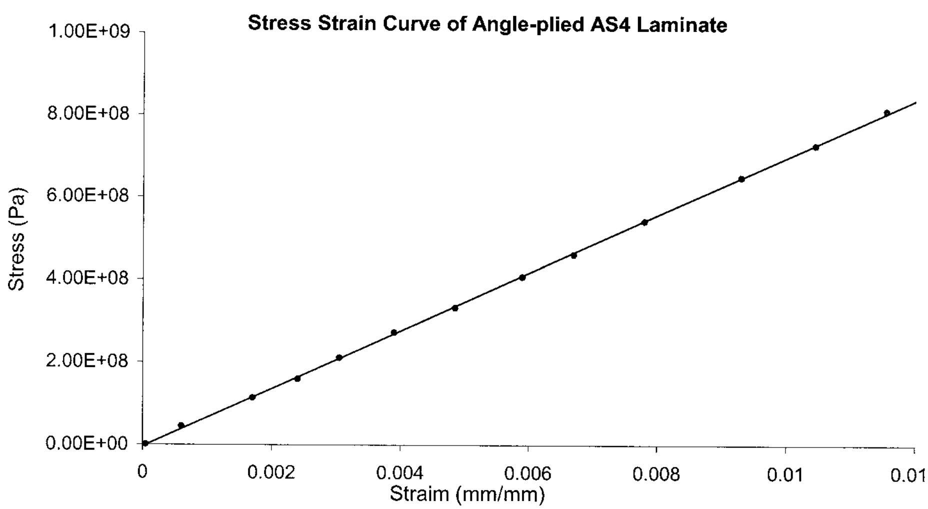

Figure 3. Stress–strain curve of angle-plied AS4.

other multidirectional laminate. The results of edge-replica process on $(0, \pm 45, 0)_s$ laminates also showed that the first crack was initiated at the same stress as that of cross-plied laminate. However, this was not the case for quasi-isotropic laminate. For quasi-isotropic laminate, first ply failure stress was observed to be less than cross-plied and angle-plied laminate. As Table 1 shows, the ultimate strengths of cross-plied and angle plied laminate were the same, whereas the ultimate strength of quasi-isotropic laminate was significantly higher. Figures 1 to 4 show the stress–strain relationship of cross-plied, unidirectional, angle-plied and quasi-isotropic laminates. The effective transverse strength of the lamina was determined using the first ply failure stress in conjunction with the classical lamination theory. Knowing the stiffness of the lamina, its effective transverse strength was calculated. As expected, because of the high degree of anisotropy, the 90° lamina is in biaxial state of stress. The biaxial strain was calculated to be 3910 $\mu\varepsilon$ and -268 $\mu\varepsilon$ in the 1 and 2 directions, respectively, and the biaxial stress was calculated as 62.7 MPa and -14.8 MPa in the 1 and 2 directions, respectively. Therefore, the ultimate effective

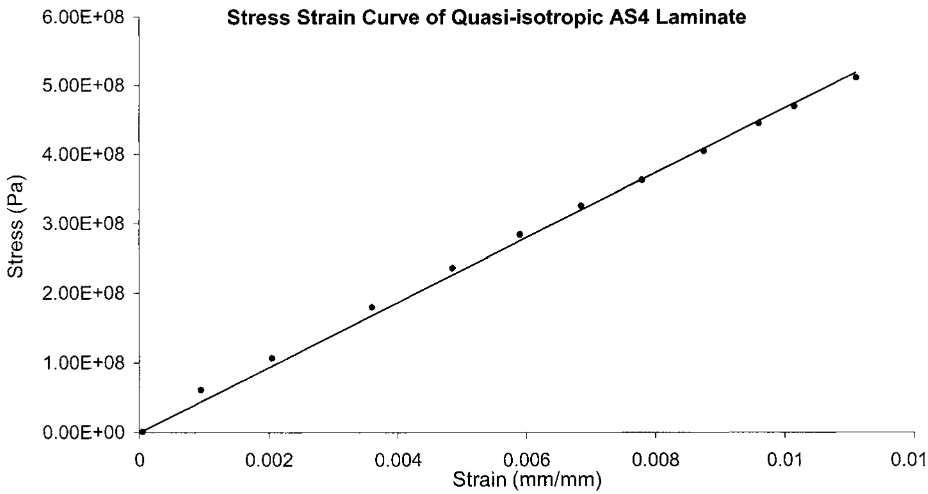

Figure 4. Stress–strain curve of quasi-isotropic AS4.

transverse strength, Y, using R of M of the laminate was determined to be 62.7 MPa, whereas 60 MPa was obtained using effective E_2 from CLT.

3.3. Experimental data for 1003

As for carbon fiber composites, the edge-replica procedure was performed on glass fiber composites. The result showed that the first crack occurred at the laminate stress of approximately 60 MPa. However, no significant change in modulus of elasticity was noticed up to 94.5 MPa. Therefore, the strength of 90° lamina was established to be 94.5 MPa. After this was established, 14 specimens were tested to 94.5 MPa stress. The longitudinal Young's modulus E_x was determined using the stress–strain linear elastic relationship at the first ply failure stress. The Young's modulus of 0° unidirectional glass fiber laminate was taken as 39 GPa, which is the published value. Using equation (2) and knowing the volume fraction of 0° and 90° is 50% each, $V_{90}^{o} = V_{0}^{o} = 50\%$, the transverse modulus was determined. At the same time, using CLT equations, average longitudinal modulus of laminate and equations (3)–(5), transverse modulus was also determined. The results are summarized below.

$E_x = 25.6$ GPa,	
$E_1 = 39.0$ GPa,	
$E_2 = 12.2$ GPa	Effective using the rule of mixture,
$E_2 = 11.8$ GPa	Effective using CLT and utilizing equations (3)–(5),
$E_2 = 8.5$ GPa	Unidirectional (by testing unidirectional coupons).

Again, there is a good agreement between rule of mixture and classical lamination theory, with approximately 3.4% difference. Transverse moduli obtained from rule

Table 2.
Effective strengths X and Y of 1003

	X (Cross-plied) $(0°, 90°)_{2s}$	X (Angle-plied) $(0, \pm 45, 0)_s$	X (Quasi-isotropic) $(0, \pm 45, 90)_s$	Effective strength Y
Experimental value	731.0 MPa	756.0 MPa	955.6 MPa	35.0 MPa unidirectional (90)
R of M prediction	748.0 MPa	750.0 MPa	837.5 MPa	43.5 MPa multi-directional
CLT prediction	747.5 MPa	748.5 MPa	824.5 MPa	42.5 MPa multi-directional

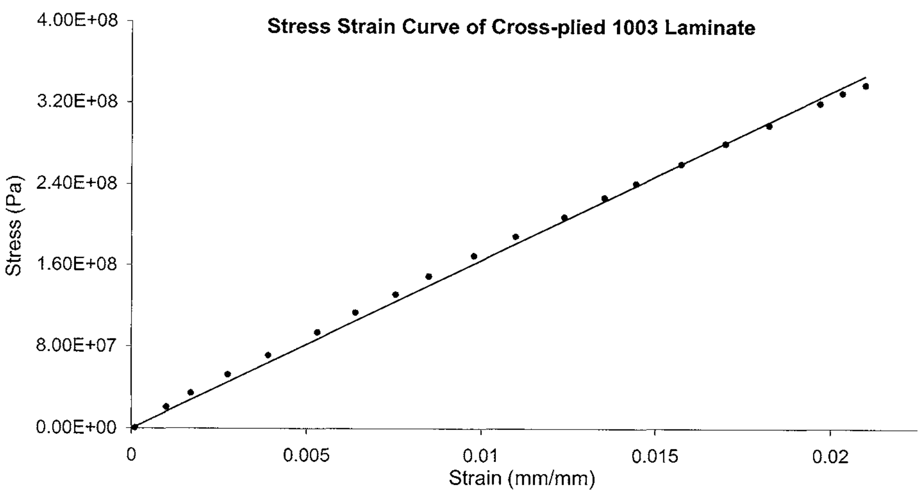

Figure 5. Stress–strain curve of cross-plied 1003.

of mixture and CLT were used to predict failure strength of cross-plied, angle-plied quasi-isotropic laminates and transverse strength Y. The values are listed in Table 2.

The ultimate strength of 0° unidirectional glass fiber laminate was recorded as 965 MPa, and the ultimate strength of cross-plied laminate was determined to be 731 MPa. In addition to cross-plied samples, 16 samples of $(0, \pm 45, 0)_s$ laminate and 8 samples of $(0, \pm 45, 90)_s$ quasi-isotropic samples were tested. The effective ultimate longitudinal strengths of cross-plied, angle-plied and quasi-isotropic laminates are tabulated in Table 2. In addition, the ultimate longitudinal strength of all samples are predicted using the procedure discussed earlier for AS4 laminates. As the results indicate, the predicted ultimate strength is in agreement with the value found experimentally. Because of the high degree of anisotropy, each lamina of the laminate is in biaxial state of stress. The failure strain was measured to be 3570 $\mu\varepsilon$ and -150 $\mu\varepsilon$ in the 1 and 2 directions, respectively, and the lamina biaxial stress was calculated as 43.5 MPa and −7.2 MPa in the 1 and 2 directions, respectively. Thus, the ultimate effective transverse strength of the laminate using the R of M was determined to be 43.5 MPa and 42.5 MPa was obtained using

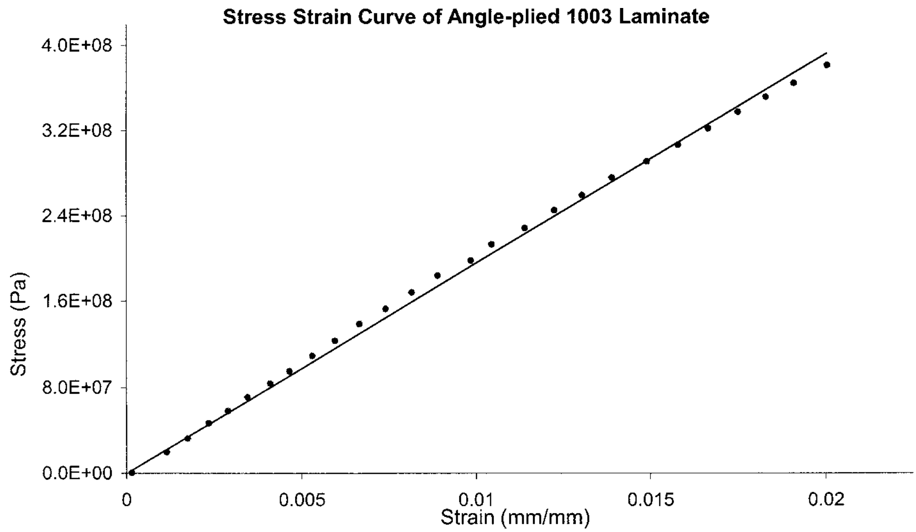

Figure 6. Stress–strain curve of angle-plied 1003.

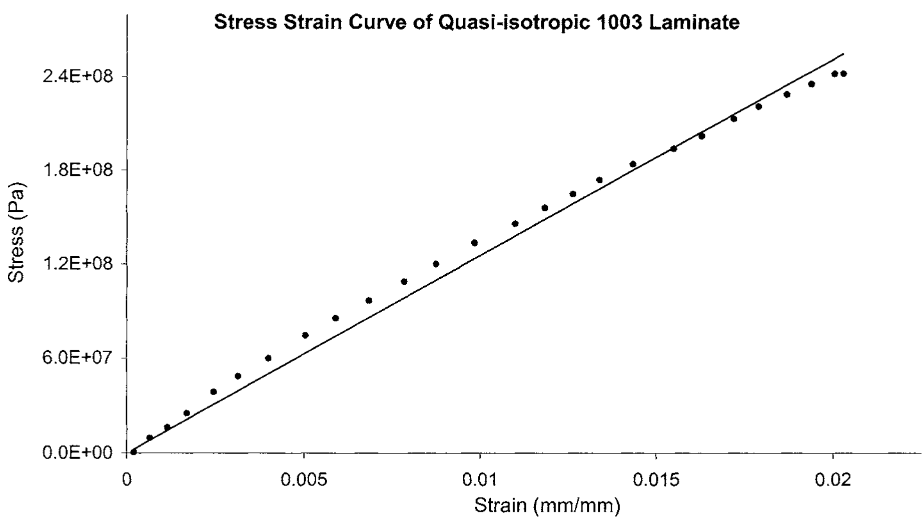

Figure 7. Stress–strain curve of quasi-isotropic 1003.

effective E_2 from the CLT. Figures 5 to 7 show the stress–strain relationship of cross-plied, angle-plied and quasi-isotropic laminates.

4. DISCUSSION OF RESULTS

The experimental results show that the transverse moduli of a unidirectional laminate for both materials are smaller than the effective moduli obtained from tests on multidirectional laminates. Indeed, the AS4 'effective' transverse modulus is about 33 percent larger than the unidirectional transverse modulus and 1003 exhibits 30 percent increase in transverse modulus. The average effective transverse modulus of the laminae in a multidirectional laminate is a function of the 'softening' that occurs as cracks initiate and spread in the laminate under loading. While the initiation of a crack would lead to failure at a low stress in a unidirectional arrangement, they are relatively insignificant in a multidirectional configuration except insofar as they affect the value of the transverse modulus. A quantitative study of the softening effect was carried out by comparing the strain developed in cross-plied with unidirectional laminates. The results indicated that for a given stress, the lamina in cross-plied arrangements strained more by 0.35% than the lamina in unidirectional configurations as depicted in Fig. 8.

The ultimate longitudinal strength of both AS4 and 1003 obtained by testing cross-plied coupons was determined to be smaller as compared to the result obtained from testing unidirectional laminates. The reduction in strength can be explained by looking at the contribution of 90° layers. Although it was argued that 90° layer would not have any contribution to the ultimate strength of the laminate, but in fact it has a functional contribution.

As has been experimentally established, the strength of fibers follows a statistical distribution. In other words, contrary to the high strength of fibers, many fibers are expected to fail at much lower stress [8]. When a crack initiates in matrix-dominated areas it propagates rapidly but the adjacent 0° layer arrests the crack. As the applied load increases, the stress at the crack tip reaches the failure stress. Since the crack cannot propagate across the fiber, the crack changes its direction and propagates parallel to the fiber direction. This causes local debonding at the interface of matrix and fiber. Since fiber failure is following a statistical distribution,

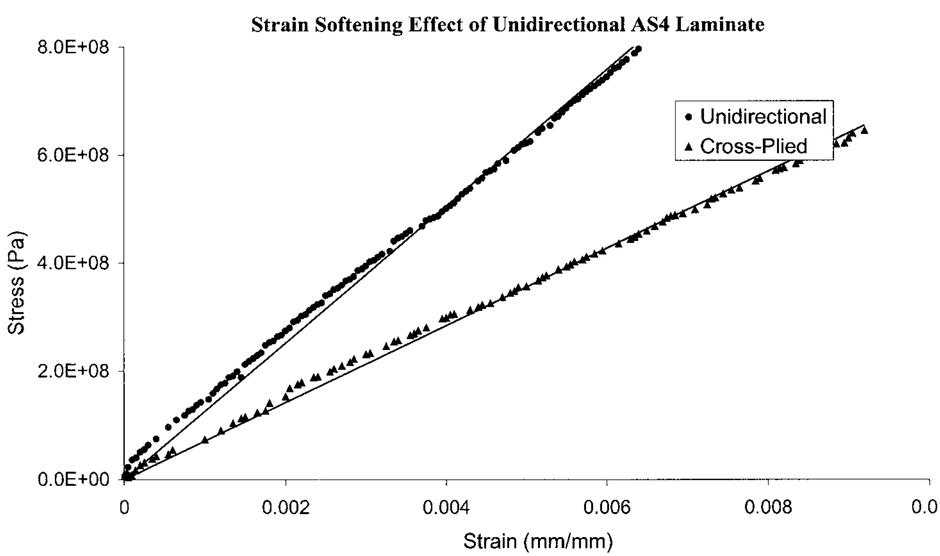

Figure 8. Softening of AS4 carbon fiber due to monotonic loading.

fibers are expected to fail locally at a lower stress than the average strength. The failure of fibers happens in a progressive manner and causes the premature failure of the laminate. As Figs 9 and 10 show, the laminate under tensile stress typically fails parallel to the fiber. Furthermore, the results for both AS4 and 1003 indicate the ultimate strength of the angle-plied test specimen corresponded to the test results of cross-plied laminates. This signifies the assumption that when the off-axis layers fail, they no longer contribute to the strength of the laminate.

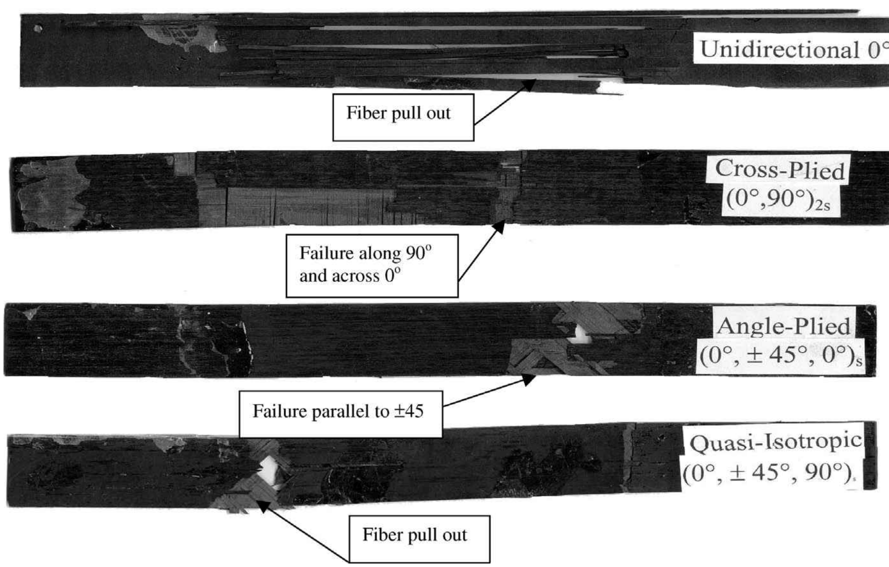

Figure 9. The failure mode of AS4.

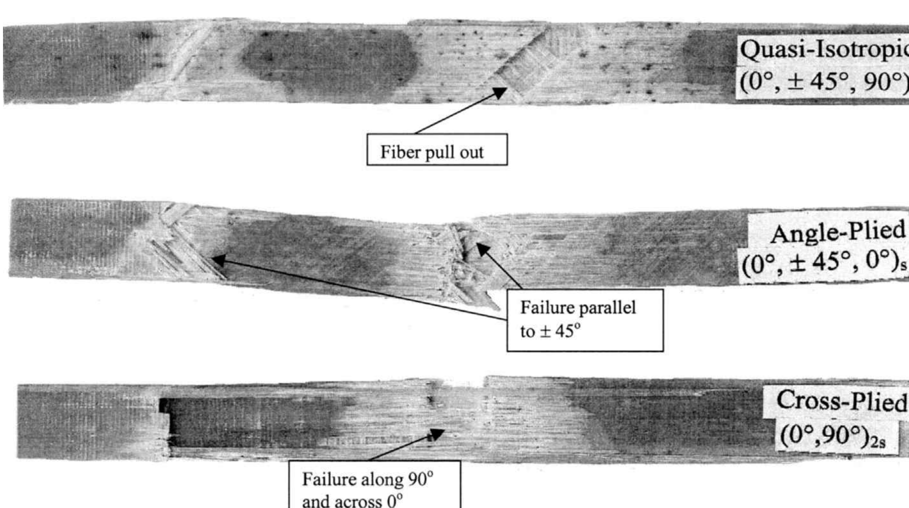

Figure 10. The failure mode of glass 1003.

Table 3.
Summary of effective properties of carbon and glass fiber epoxy

Material	$E_{2\ eff}$	$E_{2\ unidir}$	X_{eff}	X_{unidir}	Y_{eff}	Y_{unidir}
AS4	16.2 GPa	9.5 GPa	1510 MPa	1950 MPa	62.5 MPa	50 MPa
1003	12.2 GPa	8.5 GPa	731 MPa	965 MPa	43.5 MPa	35 MPa

The maximum laminate failure strain was found to be 1.15% and 2.25% for AS4 and 1003 respectively. The experimental, transverse and longitudinal moduli coupled with classical lamination theory were used to calculate the failure stresses of the laminates. The calculation was performed to find the accuracy of effective properties found in this experiment. The discrepancy resulted from the fact that CLT assumes perfect bonding of the layers, ignores the inter-laminar stresses, and also ignores the edge effect at the traction free edges of the laminate.

The test results of quasi-isotropic laminates showed that their ultimate strength was 1830MPa and 955.6 MPa for AS4 and 1003 respectively. These results are slightly less than the longitudinal strength obtained by testing unidirectional test specimens. This discrepancy could be attributed to variation in the inter-laminar stresses in multidirectional laminates. Such variations that are related to a laminate's lay-up and staking sequence are not taken into consideration in the current analysis.

5. CONCLUSION

It was argued that the unidirectional laminate properties result in erroneous failure prediction in multidirectional configuration. Therefore, the effective properties of a lamina were studied. A simple specimen and data analysis method were proposed to determine the effective properties of laminated composite. It was experimentally shown that the use of unidirectional lamina properties in predicting the failure of multidirectional laminates is inaccurate. Indeed, the effective transverse modulus for a carbon-epoxy composite was determined to be about 33% more than the unidirectional transverse modulus whereas glass-epoxy composite exhibited 30% increase in transverse modulus. The effective longitudinal strengths of the above two materials were determined to be 22.5% and 24.2% less than unidirectional properties, respectively. The effective transverse strengths were determined to be 25% and 24.3% larger than that of unidirectional transverse strength.

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